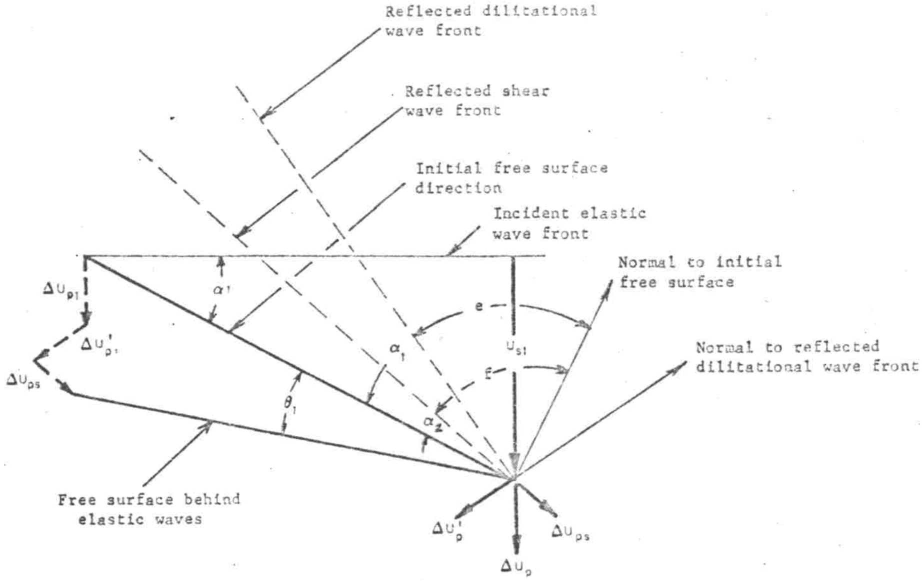

FIG. 5. Vector diagram for elastic wave interaction with the free surface.

to be reflected at the free surface. The vector diagrams in Figs. 5 and 6 show the material velocities associated with each of these waves and their relation to the free surface angles which may be measured in each experiment. In addition, in order to use Eqs. (3) and (4) to find the stress and strain behind each wave, it is necessary to determine the material velocities, U_{p1} and U_{p2}, which occur behind the first and second waves within the sample.

From Fig. 5, the free surface angle θ_1, is

$$\tan\theta_1 = \frac{(1+r_1)\cos\alpha_1 + r_2\sin\alpha_2}{1/\epsilon_1 - (1-r_1)\sin\alpha_1 - r_2\cos\alpha_2}, \quad (7)$$

where

$$r_1 = \Delta U_{p1}'/\Delta U_{p1}$$

and

$$r_2 = \Delta U_{ps}/\Delta U_{p1}$$

are the reflected material velocity ratios for the dilatational and shear waves, respectively; α_1 and α_2 are the shock front angles between the free surface and the dilatational and shear wave fronts, respectively; ϵ_1 is the strain at the yield point as defined in Eq. (2); ΔU_{p1} is the material velocity occurring behind the incident elastic wave; and $\Delta U_{p1}'$ and ΔU_{ps} are the material velocities which occur behind the reflected dilatational and shear waves, respectively, as shown in Fig. 5.

The velocity ratios are related to the angles of obliquity e and f as shown in Fig. 5 by the relationships[9–11]

$$\frac{\Delta U_{p1}'}{\Delta U_{p1}} = \frac{4\tan f\tan e - (\tan^2 f - 1)g(\nu)}{4\tan f\tan e + (\tan^2 f - 1)g(\nu)} \quad (8)$$

and

$$\frac{\Delta U_{ps}}{\Delta U_{p1}} = \frac{-4\tan e\, g(\nu)}{4\tan e\tan f + (\tan^2 f - 1)g(\nu)}, \quad (9)$$

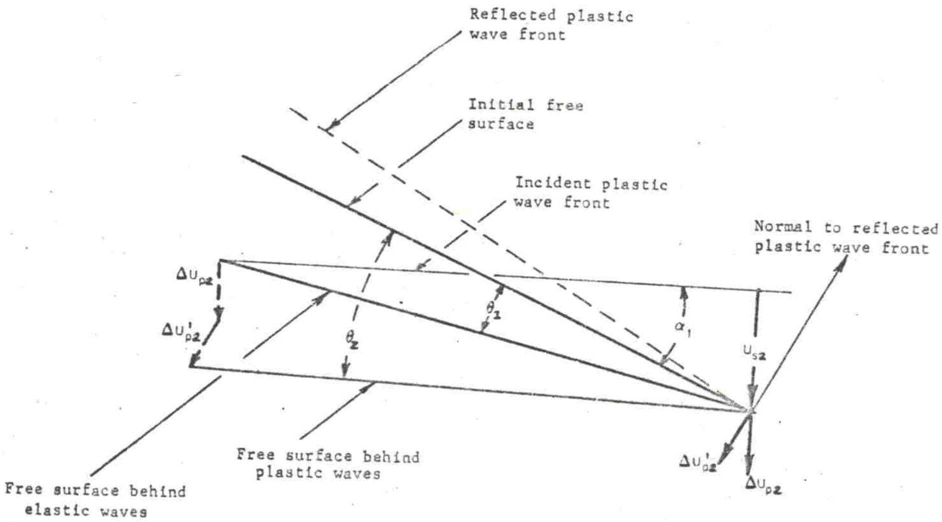

FIG. 6. Vector diagram for plastic wave interaction with the free surface.